Isotope Geochem Topic #1 Notes:
1) Radioactive Decay and Nucleosynthesis
2) Geochronology I

Reading for this topic is White Chapters 1, 2, and 4 with 3 as tangential reading.

Guide Questions:
1) What are the components of a nucleus and what keeps the nucleus together?
2) In a very general sense, which combinations of these components are “happy”? In other words, for a given number of protons, is there only one number of neutrons that gives a stable nucleus? What is the ratio of neutrons to protons in most stable nuclei (roughly)? How does the half life of the unstable nuclei change as the number of neutrons increases from the stable value(s)?
3) What particles can be ejected (or captured) to allow a nucleus to change spontaneously from an unstable arrangement of nucleons to a stable one? What convenient notation is used to express these transitions?
4) What is the name for photons that are emitted during such transitions? How is excess energy released from a nucleus after such transitions?
5) What other processes can change the nucleons in a nucleus?
6) What differential equation describes the “parent” and daughter nuclide abundances during radioactive decay of a given number of unstable parent nuclides? Why is this type of equation so common in natural science?
7) What simple equation describes how the amounts of parent and daughter nuclides evolve over time (i.e., what is the solution to the differential equation above)? What assumptions go into this simple equation? (VERY IMPORTANT)
8) What are the things that we can measure in rocks, fossils, etc. that allow us to use this equation to determine time passed?
9) What are some of the various events that can occur to “reset the radiometric clock”?
10) In cases where there are daughter isotopes inherited from the environment in which the material formed, how can we determine the amount inherited daughter isotopes and calculate the age of the material?

Radioactive decay and the physics of the nucleus

a. Number of protons (Z) determines which chemical element a nucleus is.
b. Number of neutrons determines stability for each nucleus.
c. Too few or too many makes nuclei unstable (high energy, just like an unstable compound in chemistry). Energies of nuclei are quantized, just like electronic states.
d. Decay. The nucleus moves toward a lower energy state by one of the following:
   i. Beta Decay  n \rightarrow p + e^-  (e^- = beta particle ejected),
   ii. Positron decay  p \rightarrow n + e^+  (e^+ = positron ejected),
iii. **Alpha Decay**: Ejection of 2N2P particle = alpha particle = $^4$He nucleus. This particle is heavy enough and fast enough so that the nucleus **recoils** like a gun that has shot a projectile.

iv. **Electron Capture** $p + e^- \rightarrow n$

v. **Fission**: nucleus splits into two large fragments plus free neutrons

vi. **Gamma emission** (very high energy photons, like x-rays) lower the energy of resulting fragments to get to ground states

e. **Other important nuclear processes**:

i. **Neutron capture**. Very important when there are many neutrons flying around, as in stellar interiors and pre-solar nebulae, or when we intentionally bombard a sample in a process called “neutron activation”. Neutrons can be inserted into a nucleus, possibly making it unstable. Some nuclei are very susceptible to this, others almost not at all.

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I don’t understand nuclear stability theory in great detail (e.g., the “liquid drop model”); I don’t expect you to. However, you should know that stable nuclei exist in a coherent swath (the stability valley) on a chart with proton number plotted versus neutron number. See figure 1.1 in White’s Notes. For light elements, N is about equal to Z in the stability valley. For heavier elements, N is somewhat greater than Z (e.g., N ~ 1.5Z for the heaviest elements).

Patterns to note:

- “Even-even” nuclei are more likely to be stable, odd numbers of Z or N lead to lower stability. Odd Z and odd N combinations are the least stable.
- Unstable nuclides that are close to the stability valley are slightly unstable; nuclides far away from the valley are highly unstable, with extremely short half-lives. Again, see [http://atom.kaeri.re.kr/](http://atom.kaeri.re.kr/)

**Nucleosynthesis**: This is the process of building of heavier nuclei from lighter ones, leaving the rich pattern of elemental abundances we see on earth. See figure 3.10 in White. For example, why is iron (element #26) so abundant? Why is scandium (element #21) so rare?

**The big bang** created H, He, Be, and Li, nothing heavier. Heavier nuclei were created later in successive generations of **stellar interiors and supernovae** in a series of steps.

For example, frequent neutron bombardment leads to heavier nuclides, which then decay by beta decay. The net effect is to make nuclei with more neutrons AND protons.

The processes of nucleosynthesis favor certain nuclides that are more stable and also produces more of certain nuclides that are combinations of common smaller nuclei. The processes are not fully worked out but we know they are fairly complex. See White’s notes for more detail if you want to know more about this fascinating subject.
Geochronology and Radioactive decay

Q: How does radioactive decay leave a record of time passed?
A: Evolution of the amounts of parents and/or daughters over time

HERE ARE THE “OBSERVABLES.”
- Parent abundance today
- Daughter abundance today

EQUATIONS:

First order decay: Assuming nuclei decay randomly, the change in the number of parent isotopes per unit time depends on the number present, multiplied by some constant. Translating this common-sense statement into mathematical terms, we get the following differential equation:

\[ \frac{dN}{dt} = -\lambda N \]  

Integrate this to get a solution:

\[ N = N_0 e^{\lambda t} \] or \[ \ln \left( \frac{N}{N_0} \right) = \lambda t \]  

so… \[ t = \frac{1}{\lambda} \ln \left( \frac{N}{N_0} \right) \]  

Now think about our “observables”, the things we can measure today. One of those is daughter nuclides... \( D^* \) is the amount of a daughter created by radioactive decay:

\[ D^* = N_0 N = N(e^{\lambda t} - 1) \]  

But we measure the total daughter abundance, \( D \), which usually includes some inherited daughter, \( D_0 \):

\[ D = D_0 + N(e^{\lambda t} - 1) \]  

- Decay constants, \( \lambda \)’s, can be looked up in reference books.
- **Major assumption**: When we write equations for parent decay and daughter ingrowth, we are assuming CLOSED SYSTEMS. Think about what happens if atoms are free to come and go!!! If so, we would expect “contamination” of our “clock”, or leakage of the decay products.

SO..., we can calculate age using…. \[ t = \frac{1}{\lambda} \ln \left( \frac{D(t)}{D(0)} \right) + 1 \] (6)

Or we can use \[ t = \frac{1}{\lambda} \ln \left( \frac{N}{N_0} \right) \] (7), if we happen to know what \( N_0 \) was.
BUT HOW DO WE DETERMINE D₀ or N₀?

different systems have different events that reset the radiometric clocks:
1) Crystallization of a rock from magma
2) Recrystallization during metamorphism
3) Cooling of crystals below a “closure temperature”
4) Growth of crystals from fluid
5) Separation of magma from the mantle
6) Initial exposure to cosmic rays
7) Last exposure to cosmic rays
8) Last exposure to cosmogenic nuclides in the atmosphere (groundwater)
these are events that can “reset the isotopic clock” in some way, and give us a way of knowing D₀ or N₀.

1) Some dating methods involve knowing N₀.
   Example: ¹⁴C age dating. We know that ¹⁴C is produced in the atmosphere at a “known” rate (more on this later: it varied in the past). Therefore, we “know” how much ¹⁴C was in each gram of carbon derived from the atmosphere. You can see that we know something about N₀ and can then use equation (7) to get age if we measure the amount of ¹⁴C in each gram of carbon in the sample today.

2) With other dating methods we don’t know what N₀ was…. A difficult part of geochronology, as you would expect, is determining what the starting condition was, possibly billions of years ago.

Procedure: we measure D(t) and N(t), and we look up []. We need to get at D₀ somehow. What other info can we get out of the rock, aside from just D(t) and N(t) from one crystal?
   • If we’re lucky, we can find a crystal where D₀ is very close to zero. This is often the case with ⁴⁰K-⁴⁰Ar age dating and U-Pb dating of zircons (later).
   • If all crystals have inherited daughter, we can measure N and D on another mineral and then calculate the amount of inherited daughter. This is the “Isochron method”. Plot a collection of different minerals on an isochron plot like the one below. ⁸⁷Rb decays to ⁸⁷Sr; ⁸⁶Sr is a stable reference isotope.

\[
\frac{[87\text{Sr}]}{[86\text{Sr}]} \quad \frac{[87\text{Rb}]}{[86\text{Sr}]} \]
1) The slope of the line is related to time passed since “closure”; \( t = \frac{1}{l} \ln(Slope + 1) \). See appendix below for details if you are interested.

2) IMPORTANT ASSUMPTION: All minerals inherited the same \([^{87}\text{Sr}] / [^{86}\text{Sr}]\) ratio at the time of closure. Note the intercept of the plot gives the initial \([^{87}\text{Sr}] / [^{86}\text{Sr}]\) ratio.

3) Why do we plot ratios instead of just concentrations? Because when we normalize the parent and daughter abundances to \(^{86}\text{Sr}\), this removes problems caused by the fact that each mineral has its own tendency to take in or exclude \text{Sr} (and Rb) as it grows.

\[
\frac{^{87}\text{Sr}}{^{86}\text{Sr}} = \frac{^{87}\text{Sr}}{^{86}\text{Sr}} + \frac{^{87}\text{Rb}}{^{86}\text{Sr}} \left( e^{\delta t} - 1 \right)
\]

In practice, we do the following:

1) Obtain data and create a plot like the one above.
2) Fit a straight line to the data. Multiple point isochrons can yield an estimate of uncertainty- see White Lecture 5 notes. To get the most accurate result, you should incorporate measurement uncertainties in both x and y directions! i.e., use a better statistical approach than simple linear regression.
3) Calculate the slope to get age.
4) Calculate the intercept to get \( R_0 \).

**Appendix 1: Derivation of the isochron equations:**

Relationship between plot slope and time:
From before we have... \( D = D(0) + N(e^{\delta t} - 1) \) ... now divide by the denominator isotope to get an isotope ratio….

\[
R = R(0) + R_{PD}(e^{\delta t} - 1)
\]

this is a linear equation on the isochron plot:
\[
y = b + x m
\]

We have two equations like this for our two minerals. Two equations in two unknowns (\( R(0) \) and \( t \))

Eliminate \( R(0) \) by subtracting the equation for one mineral from that of the other:
\[
R = R_{PD}(e^{\delta t} - 1)
\]

Solve for \( t \):
\[
t = \frac{1}{l} \ln\left( R / R_{PD} + 1 \right) = \frac{1}{l} \ln(Slope + 1)
\]

Solve for \( R(0) \):
\[
R(0) = R - R_{PD} \ast \frac{R}{R_{PD}} = R - R_{PD} \ast \text{Slope}
\]