Why is bed shear stress important?

Provides an index of fluid force per unit area on the stream bed, which has been related to sediment mobilization and transport in many theoretical and empirical treatments of sediment transport.
Calculation of Bed Shear Stress

- Various methods based on
  - Reach-averaged relations
  - Theoretical assumptions about structure of turbulence
  - Direct measurements of turbulence
Reach-Averaged Method

Mean Bed Shear Stress - force per unit area exerted by a “block” of water on the channel boundary as it moves downstream

\[ F = \gamma W D X \sin \theta \ (N) \quad [\text{MLT}^{-2}] \]

\[ \tau = \gamma R S \ (N \ m^{-2}) \]

(downstream oriented component of the weight of the block)
Reach-Averaged Method

Advantages -

Serves as an index of the total resistance by ALL frictional influences on the flow (particle-, bedform-, bar-, and planform-scale effects)

Relatively easy to measure

Disadvantages -

Does not provide information on spatial variation in resistance at sub-reach scale

Is not necessarily a good index of the competence of the stream to move sediment
“Law of the Wall” Method

- Based on the assumption that the velocity profile in the lower portion (15-20%) of an open channel flow has a logarithmic structure:
"Law of the Wall" Method

\[ u = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \]

\( u \) = mean velocity (in vertical), \( u_* \) = shear velocity, \( \kappa \) = von Karman’s constant, \( z \) = distance above bed, \( z_0 \) = roughness height (height above bed where velocity goes to zero)
“Law of the Wall” Method

\[ u = \frac{u^*}{K} \ln \left( \frac{z}{z_0} \right) = \frac{u^*}{K} \ln z - \frac{u^*}{K} \ln z_0 = m \ln y + b \]

where

\[ m = \frac{u^*}{K}, \quad b = -\frac{u^*}{K} \ln z_0 = -m \ln z_0 \]
“Law of the Wall” Method

Measure mean velocities \( (u) \) at various heights above bed in lower 15-20% of the flow

Regress the values of \( u \) against the logarithms of \( z \) to get estimates of \( m \) and \( b \)

Calculate values of shear velocity, bed shear stress, and roughness height

\[
\begin{align*}
  u_* &= \kappa m, \\
  \tau &= \rho u_*^2, \\
  Z_0 &= e^{-b/m}
\end{align*}
\]
"Law of the Wall" Method

Advantages

Provides local measure of shear stress
Can be used to map spatial patterns of shear stress and roughness height at subreach scale
Standard error of estimate of regression can provide an estimate of error in $u_*$

Disadvantages

Flow must conform with logarithmic velocity profile
Errors in measurement of $u$ and $z$ can influence results (least precise of "law of wall" methods)
Variants on “Law of the Wall”

\[ u = \frac{u_*}{\kappa} \ln \left( \frac{z}{ad_p / 30} \right) \]

\[ a = 3, \ p = 84 \text{ Whiting and Dietrich, 1990} \]

\[ a = 2.85, \ p = 90 \text{ Wilcock et al. 1996} \]

\( d_p = \text{particle size for which } p\% \text{ of material is finer} \)

Advantage

requires only a single near-bed velocity reading in lower 20% of flow for estimate of \( u_* \)

Disadvantage

requires information on the grain-size distribution of bed material

Applies to gravel-bed rivers only and assumes that empirical relation \( z_0 = (ad_p/30) \) applies to all such rivers
Variants on “Law of the Wall”

\[ U = \frac{u_*}{\kappa} \ln \left( \frac{h}{e(ad_p / 30)} \right) \]

- \( U = \) depth-averaged velocity
- \( h = \) flow depth
- \( e = \) base of natural logarithms

**Advantage**
Has less variability than other “law of the wall” methods

**Disadvantage**
Requires measurement of velocity profile to determine mean [could perhaps be used with a single measure of \( U \) (6/10th depth)]
Evaluation of “Law of the Wall”
Precision (Wilcock, 1996)

Lowest precision – slope of velocity profile

Highest precision – depth-averaged velocity

Says nothing about accuracy of the various methods

Figure 1. Differences between replicate calculations of $u_*$ for three different methods: (a) slope of velocity profile (1) in lower half of flow, (b) a single velocity observation in the lower 20% of the flow and (1), and (c) depth-averaged velocity and (2). Difference in $u_*$ plotted as a function of mean $u_*$ for each replicate pair. Differences do not depend strongly on magnitude of $u_*$. Standard errors $\sigma(u_*)$ are calculated from (3).
Direct Measurement:
Near-bed Reynolds Shear Stress

\[ \tau_b = -\rho u'_b w'_b \]

\[ u'_b = \text{near - bed downstream velocity fluctuation} \]

\[ w'_b = \text{near - bed vertical velocity fluctuation} \]
Direct Measurement: Near-bed Reynolds Shear Stress

**Advantage**

Direct measurement of turbulent shear stress near the bed

**Disadvantage**

How close to the bed do you need to be? (seems to depend on roughness characteristics and purpose of measurement)

Many measurement devices cannot measure velocity fluctuations accurately close to the bed

Need 2-D measurements of turbulent fluctuations
Turbulent Kinetic Energy Method

\[ TKE = k = 0.5(u'^2 + v'^2 + w'^2) \]
\[ \tau_b = C_1 \rho k \]
\[ C_1 \approx 0.19 \]

Alternative Formulation

\[ \tau_b = C_2 \rho w'^2 \]
\[ C_2 = 0.9 \]
Turbulent Kinetic Energy Method

Advantages
No need to estimate roughness height
Single near-bed reading of 3-D velocities

Disadvantages
How close to bed
3-D velocity measurements
Values of $C_1$ and $C_2$ not derived from streams or rivers (oceans)
Is Bed Shear Stress the Right Index?

Some recent studies have questioned whether looking at turbulence fluxes or velocity profiles is the right approach for understanding sediment transport. Instead, look at actual sediment mobilization and transport and relate it empirically to various velocity measures, including *instantaneous* velocities:

\[ u = \langle u \rangle + u' \]
References


Turbulent boundary layer structure - the influence of roughness
The influence of roughness

- a. Smooth
- b. Perforated plate
- c. Sand grain
- d. Wire screen

Rough-wall velocity profiles (Bergstrom et al., 2002)
Rough-wall velocity profiles: scale by freestream velocity (outer-wall scaling) (Bergstrom et al., 2002)
Rough-wall velocity profiles: scale by inner-wall variables (Bergstrom et al., 2002)

Still great debate on scaling (inner/outer) and also if roughness effects can be collapsed
The influence of roughness density (plan area of elements: total area)

Region 1: TI decreases linearly to surface and $u_*$ is good to collapse profiles

Region 2: TI approx. constant at approx. 2

Region 3: roughness density influential

Nowell and Church, JGR, 1979
The influence of roughness density

After Nowell and Church, JGR, 1979
The influence of roughness on turbulence intensity (Grass 1971)
Influence of roughness on turbulence intensity (Ligrani and Moffat 1986)
Model of smooth wall TBL structure (Smith, 1996)
Model of rough wall TBL structure (Smith, 1996)
Links to Large-Scale-Motions (Falco, 1977) and seminars
Links to Sediment Entrainment (Grass, 1971)
References


Research Projects

• The flow dynamics of idealized interacting gravel particles (x2)
• Turbulence in transitional flows: the influence of fine sediment on flows (x2)
• Mean flow and turbulence over a dune field in the Missouri River
• Flow over the ripple:dune transition – turbulence and vorticity
• Flow separation at confluences (x2)
• The flow dynamics of bendway weirs (x2)
• Flow structure and turbulence within meander bends (x2)

• Your possible ideas for projects?...we are very keen to encourage projects with your own data or develop your own ideas for a topic, which can be worked into this format.....